Enrollment No:-__________ Exam Seat No:-_______

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: 5SC02MTC1 **Subject Name: Differential Geometry**

Course Name: M.Sc. (Mathematics) Date: 18/5/2015

Semester:II Marks:70

Time:10:30 TO 01:30

Instructions:

1) Attempt all Questions of both sections in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

Section– I

Q-1 a) Find arc length for the curve $\bar{r} = (a\cos t\,, a\sin t, ct)$. (02)

b) Define circle of curvature. Write equation of circle of center of curvature. (02)

c) Find tangent plane and normal line for the surface $xyz = a^3$. (02)

d) State Serret – Frenet equations. (01)

Q-2 a) Define spherical indicatrix of tangent. Find tangent, principal normal, binormal curvature and torsion for the spherical indicatrix of tangents of the arc length parameterized curve $\bar{r}$. (07)

b) Let $\bar{r}(t)$ be a curve and $\dot{r}(t) \neq 0$ for all t, prove that curvature $k = \frac{|\dot{r} \times \ddot{r}|}{|\dot{r}|^3}$. (07)

OR

Q-2 a) Define Involute. Also find tangent, principal normal, binormal, curvature and torsion of Involute of a given curve. (07)

b) Find curvature and torsion of the curve $\bar{r} = (u - sinu, 1 - cosu, 3)$. (07)

Q-3 a) Define Bertrand curves. Also for that prove $\tau\tau_1$ = constant. (06)

b) Derive necessary condition for surface $z = f(x, y)$ representing a developable surface. (04)

c) Prove that the envelope of the family of paraboloids $x^2 + y^2 = 4a\,(z - a)$ is $x^2 + y^2 = z^2$. (04)

OR

Q-3 a) Prove that a curve is uniquely determined except as to position in space when its curvature and torsion are given functions of it's arc length. (06)

b) Prove that the edge of regression for the Osculating developable is the curve itself. (04)

c) Prove that the surface $xy = (z-c)^2$ is a developable. (04)

Section – II

Q-4 a) Define conjugate directions. (02)

b) Define null line. Write conditions for parametric curves will be null line. (02)

c) State Meunier's theorem. (02)

d) Write equation for principal direction. (01)

Q-5 a) Define normal curvature k_n of a surface. Prove that $k_n = \frac{L\,du^2+2M\,dudv+N\,dv^2}{E\,du^2+2F\,dudv+Gdv^2}$ (06)

b) Find first fundamental form for the surface $2z = \frac{x^2}{a} + \frac{y^2}{b}$. (04)

c) Prove that the necessary and sufficient condition for lines of curvature to be parametric curve is $F = 0 = M$. (04)

OR

Q-5 a) State and prove Euler's theorem for normal curvature. (06)

b) Find second order magnitude for $\bar{r} = (u\cos\emptyset, u\sin\emptyset, \emptyset)$. (04)

c) Find Gaussian and mean curvature for the surface $\bar{r} = (u\cos\theta, u\sin\theta, f(u))$. (04)

Q-6 a) Prove that christoffel symbol of first kind $\Gamma_{\text{ijk}} = \frac{1}{2}\left(\frac{\partial g_{jk}}{\partial u^i} + \frac{\partial g_{ki}}{\partial u^j} - \frac{\partial g_{ij}}{\partial u^k}\right)$. (06)

b) Find equation of asymptotic line and their torsion for the surface generated by tangent at a given twisted curve. (04)

c) Find the parabolic points on the surface $z = x^3 - y^3$. (04)

OR

Q-6 a) Prove that the first and second fundamental quantities satisfy $\frac{\partial b_{11}}{\partial u^2} + \Gamma_{11}^{l} b_{l2} = \frac{\partial b_{12}}{\partial u^1} + \Gamma_{12}^{l} b_{l1}$ and $\frac{\partial b_{21}}{\partial u^2} + \Gamma_{21}^{l} b_{l2} = \frac{\partial b_{22}}{\partial u^1} + \Gamma_{22}^{l} b_{l1}$ (06)

b) Find Christoffel symbols of the first kind for the surface generated by principal normal at a given twisted curve. (04)

c) Prove that all the points on the surface $x^2 + y^2 + z^2 = a^2$ are elliptic points. (04)

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